

IFT-UAM/CSIC-99-1
hep-th/9901078
January 18th, 1999

The D8-Brane Tied up: String and Brane Solutions in Massive Type IIA Supergravity

Bert Janssen^{♣♣}¹, Patrick Meessen[♠]² and Tomás Ortín^{♣♣}³

[♠] *Instituto de Física Teórica, C-XVI, Universidad Autónoma de Madrid
E-28049-Madrid, Spain*

[♣] *I.M.A.F.F., C.S.I.C., Calle de Serrano 113 bis
E-28006-Madrid, Spain*

Abstract

We present two new solutions of Romans' massive type IIA supergravity characterized by the two non-trivial massive potentials of Romans' theory: the NSNS 2-form and the RR 7-form. They can be interpreted respectively as the intersection of a fundamental string and a D8-brane over a D0-brane and the intersection of a D6-brane with a D8-brane over a NSNS5-brane. The D8-brane manifests itself through the mass parameter and in the massless limit one recovers the standard fundamental string and D6-brane solutions.

Although these solutions do not have the usual form for BPS bound states at threshold and each of them involves 3 objects, both of them preserve 1/4 of the supersymmetries.

¹E-mail: Bert.Janssen@uam.es

²E-mail: Patrick.Meessen@uam.es

³E-mail: tomas@leonidas.ima.iff.csic.es

Introduction

Supergravity theories (and specially SUEGRAs⁴) are the low-energy effective actions of string theories and therefore harbor fields corresponding to the string massless modes. In particular they contain $p + 1$ -form potentials whose sources can be identified with perturbative and non-perturbative string theory p - and $\tilde{p} = d - p - 4$ -dimensional objects: the fundamental string, the solitonic 5-brane and the Dirichlet Dp -branes.

Type IIA superstring theory contains Dp -branes with $p = 0, 2, 4, 6, 8$ [1]. However, the standard type IIA SUEGRA [2, 3, 4] only contains RR potentials for the first four cases. It was later realized [5, 6] that since an 8-brane couples to a 9-form potential which can be dualized into a constant parameter (“1-form field strength”), the low-energy theory of the type IIA superstring is Romans’ massive type IIA theory [7], until then considered an exotic deformation of the standard theory.

Romans’ theory contains a constant parameter m with dimensions of mass which can be interpreted as the Hodge dual of the 10-form field strength associated to a D8-brane [6]. The D8-brane carries no dynamical degrees of freedom and should be interpreted as a background for the type IIA theory. This is however very non-trivial background: the parameter not only appears in the action through a kinetic-like term

$$\int d^{10}x \sqrt{|g|} \left[-\frac{1}{2} m^2 \right] ,$$

with the (in)dependence of the dilaton characteristic of RR potentials (in the string frame), but also appears as a mass parameter for the NSNS 2-form.

Romans’ massive type IIA theory has many mysterious features. In particular, its 11-dimensional origin (and that of the D8-brane) cannot be the standard 11-dimensional supergravity and one has to appeal to non-covariant generalizations associated to backgrounds with special isometric directions [8, 9, 10] (see also [11, 12]).

The physical reason why the NSNS 2-form B gets a mass in this background is not well understood either although this is the most notorious feature of this theory, perhaps because it has not been fully appreciated. Actually, the mass term for B has been ignored in the search for supersymmetric solutions describing the intersection of fundamental strings or solitonic 5-branes with D8-branes⁵.

Furthermore, in Ref. [8] a previously ignored mass term for the RR 7-form potential $C^{(7)}$ was shown to occur.

In this letter we will present two supersymmetric solutions of massive type IIA SUEGRA for which these two mass terms do not vanish. The first solution which we will call *massive string* has a non-trivial B field (plus metric and dilaton). It preserves 1/4 of the supersymmetries and contains an arbitrary harmonic function allowing for the description of several objects in equilibrium. In the massless limit it becomes the fundamental string solution [16] and it will be interpreted as the intersection of a fundamental string with a D8-brane, the former occupying the only direction orthogonal to the latter. It can be argued

⁴SUper-Extended-GRavity theories.

⁵M-brane and Dp -brane intersections were first discussed in Refs. [13, 14, 15].

that there is a D0-brane present at the intersection. The existence of this configuration was demonstrated in Ref. [17].

The second solution, which we will call *massive D6-brane* has a nontrivial $C^{(7)}$ field (plus metric and dilaton). Consistency of the equations of motion requires the presence of a non-trivial B . This solution also preserves 1/4 of the supersymmetries and contains an arbitrary harmonic function allowing for the description of several objects in equilibrium. In the massless limit the B field vanishes and the solution becomes the D6-brane solution and it will be interpreted as the intersection of a D6-brane with a D8-brane over 5 spacelike directions. However, the presence of the B field in the massive case will be interpreted as a solitonic 5-brane living in the intersection.

1 Massive Type IIA SUEGRA

Here we give the bosonic equations of motion and the fermionic supersymmetry transformation laws of massive type IIA SUEGRA in the string frame including the dual RR and NSNS potentials. Many of the general expressions are also valid for the IIB theory. Due to the explicit occurrence of potentials in the action, they can only be dualized on-shell. The dual potentials are defined by the relations between field strengths

$$\begin{cases} G^{(10-n)} &= (-1)^{[n/2]\star} G^{(n)}, \\ H^{(7)} &= e^{-2\phi\star} H, \end{cases} \quad (1)$$

plus the Bianchi identities

$$\begin{cases} dG - H \wedge G &= 0, \\ dH &= 0, \\ dH^{(7)} + \frac{1}{2}\star G \wedge G &= 0, \end{cases} \quad (2)$$

and the equations of motion

$$\begin{cases} d\star G + H \wedge G &= 0, \\ d(e^{-2\phi\star} H) + \frac{1}{2}\star G \wedge G &= 0, \\ d(e^{2\phi\star} H^{(7)}) &= 0, \end{cases} \quad (3)$$

where we are using the notation [18, 19, 8] in which which forms of different degrees are formally combined into a single entity:

$$\begin{cases} C &= C^{(0)} + C^{(1)} + C^{(2)} + \dots, \\ G &= G^{(0)} + G^{(1)} + G^{(2)} + \dots. \end{cases} \quad (4)$$

These expressions are valid both for the type IIB and for the massive type IIA theory if one selects respectively odd and even rank and odd rank RR differential form field strengths and one makes the identification

$$G^{(0)} = m. \quad (5)$$

The field strengths that correspond to these Bianchi identities and equations of motion are given by

$$\begin{cases} G &= dC - H \wedge C + me^B, \\ H^{(7)} &= dB^{(6)} - mC^{(7)} - \frac{1}{2} \sum_{n=1}^{n=4} \star G^{(2n+2)} \wedge C^{(2n-1)}, \\ \mathcal{H}^{(7)} &= d\mathcal{B}^{(6)} + \frac{1}{2} \sum_{n=1}^{n=4} \star G^{(2n+3)} \wedge C^{(2n)}, \end{cases} \quad (6)$$

where calligraphic fields belong to the IIB theory.

These equations have to be supplemented by the dilaton equation of motion

$$R + 4(\partial\phi)^2 - 4\nabla^2\phi + \frac{1}{2!}H^2 = 0, \quad (7)$$

and the Einstein equation of motion (where we have already eliminated R with the use of the dilaton equation of motion)

$$R_{\mu\nu} - 2\nabla_\mu \nabla_\nu \phi + \frac{1}{4} H_\mu^{\rho\sigma} H_{\nu\rho\sigma} - \frac{1}{4} e^{2\phi} \sum_n \frac{(-1)^n}{(n-1)!} T^{(n)}_{\mu\nu}, \quad (8)$$

where $T^{(n)}_{\mu\nu}$ are the energy-momentum tensor of the RR fields:

$$T^{(n)}_{\mu\nu} = G^{(n)}_{\mu}{}^{\rho_1 \dots \rho_{n-1}} G^{(n)}_{\nu\rho_1 \dots \rho_{n-1}} - \frac{1}{2n} g_{\mu\nu} G^{(n)2}, \quad (9)$$

and, in particular

$$T^{(0)}_{\mu\nu} = -\frac{1}{2} m^2 g_{\mu\nu}. \quad (10)$$

These equations are also valid both for the type IIA and IIB theories. Observe that the contributions of the energy-momentum tensors of dual fields add up, except in the $n = 5$ case.

Let us now focus on the (massive) type IIA theory. The supersymmetry transformation law for the gravitino and dilatino are⁶

$$\begin{cases} \delta_\epsilon \psi_\mu &= \left\{ \partial_\mu - \frac{1}{4} \left(\varphi_\mu + \frac{1}{4} \Gamma_{11} \not{H}_\mu + \frac{1}{2 \cdot 7!} e^{2\phi} \Gamma_{\mu\nu_1 \dots \nu_7} H^{(7)}{}^{\nu_1 \dots \nu_7} \right) \right\} \epsilon \\ &+ \frac{i}{16} e^\phi \sum_{n=0}^{n=4} \frac{1}{(2n)!} \not{G}^{(2n)} \Gamma_\mu (-\Gamma_{11})^n \epsilon, \\ \delta_\epsilon \lambda &= \left[\not{\partial} \phi + \frac{1}{4} \left(\frac{1}{3!} \Gamma_{11} \not{H} - \frac{1}{7!} e^{2\phi} \not{H}^{(7)} \right) \right] \epsilon + \frac{i}{8} e^\phi \sum_{n=0}^{n=4} \frac{5-2n}{(2n)!} \not{G}^{(2n)} (-\Gamma_{11})^n \epsilon. \end{cases} \quad (11)$$

⁶We work with real 32-component ad purely imaginary gamma matrices satisfying $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$ where η^{ab} has mostly minus signature. Finally, $\Gamma_{11} = -\hat{\Gamma}^0 \dots \hat{\Gamma}^9$.

As explained in the introduction, the mass parameter occurs in the form of a cosmological constant (or, in the Einstein frame, of an unbound potential for the dilaton). Furthermore, the field strengths of $C^{(1)}$ and $B^{(6)}$ contain the terms

$$\begin{cases} G^{(2)} &= dC^{(1)} + mB, \\ H^{(7)} &= dB^{(6)} - mC^{(7)} + \dots \end{cases} \quad (12)$$

associated to these terms there are two *massive gauge transformations*

$$\begin{cases} \delta C^{(1)} &= -m\Lambda^{(1)}, \\ \delta B &= d\Lambda^{(1)}, \end{cases} \quad \begin{cases} \delta B^{(6)} &= m\Lambda^{(6)}, \\ \delta C^{(7)} &= d\Lambda^{(6)}, \end{cases} \quad (13)$$

using which one can completely eliminate $C^{(1)}$ and $B^{(6)}$ everywhere. Then, the kinetic terms of these Stückelberg fields become mass terms for B and $C^{(7)}$ respectively. Only these two terms are massive.

Now, solutions describing p -branes in massive type IIA SUEGRA are automatically solutions describing the intersection of those p -branes with a D8-brane associated to the mass parameter. General solutions for the intersection of p_1 and p_2 branes have been found in the literature using a generic model whose action contains only kinetic terms for the dilaton and the $(p_1 + 2)$ - and $(p_2 + 2)$ -form field strengths. Therefore, those solutions can potentially describe correctly intersections involving a D8-brane and a D0-, D2- and D4-brane, associated to massless fields. However, they cannot correctly describe the intersections of a D8-brane and a fundamental string, a solitonic 5-brane or a D6-brane. Study of the supersymmetry algebra reveals that these solutions should exist and preserve 1/4 or the supersymmetries [17]. In the next two sections we will present the corresponding solutions and will comment on some of their unusual features.

2 Massive String

This solution is given by

$$\begin{cases} ds^2 &= \Omega^{-1} (dt^2 - dy^2) - d\vec{x}_8^2, \\ B_{\underline{ty}} &= \pm (\Omega^{-1} - 1), \\ C^{(1)}_{\underline{t}} &= \pm my, \\ e^{-2\phi} &= \Omega, \end{cases} \quad (14)$$

where

$$\left\{ \begin{array}{l} \vec{x}_8 = (x^1, \dots, x^8) = (x^m), \\ \partial_{\underline{m}} \partial_{\underline{m}} \Omega = -m^2, \\ \partial_{\underline{y}} \Omega = \alpha m. \end{array} \right. \quad (15)$$

This solution has the following properties:

1. The function Ω consists of three pieces: a piece linear in y (which is interpreted as the coordinate along the string and perpendicular to the D8-brane), a piece quadratic in \vec{x}_8 (which are interpreted as the worldvolume coordinates of the D8-brane, orthogonal to the string) and a harmonic function of \vec{x}_8 :

$$\Omega = \alpha m y - \sum_p M_p x^p x^p + H(\vec{x}_8), \quad \sum_p M_p = \frac{1}{2} m^2, \quad \partial_{\underline{m}} \partial_{\underline{m}} H = 0. \quad (16)$$

Thus, it can describe, in principle, several objects in equilibrium.

2. In the massless limit $\Omega(y, \vec{x}_8) = H(\vec{x}_8)$ and for the right choice of H it is just the fundamental string solution [16].
3. The limit in which the string is eliminated is unattainable from this solution. Even if we set $H = 0$ Ω is still non-trivial and the solution will have only 1/4 of the supersymmetries unbroken.
4. The $C^{(1)}$ field can be completely gauged away, canceling the ∓ 1 in $B_{\underline{t}\underline{y}}$. We have introduced it in order to have $B_{\underline{t}\underline{y}}$ in the form which corresponds to a fundamental string source. (It can be argued that there is a D0-brane in the intersection between the string and the D8-brane, as we will see when we study the unbroken supersymmetry the solution.)
5. We have a solution for any value of the constant α . However, only for $\alpha = \mp 1$ the solution is supersymmetric. This is a quite unusual behavior

Let us now find the unbroken supersymmetries. We will only analyze the dilatino supersymmetry rule to show how it works. In this case

$$\delta_\epsilon \lambda = \left(\not{\partial} \phi + \frac{1}{2 \cdot 3!} \Gamma_{11} \not{H} \right) \epsilon + \frac{5i}{4} m e^\phi \epsilon - \frac{3i}{8} e^\phi \not{G}^{(2)} \Gamma_{11} \epsilon, \quad (17)$$

with

$$\left\{ \begin{array}{l} \not{H} = \mp 3! \partial_{\underline{m}} \Omega \Gamma^m \Gamma^{0y}, \\ \not{\partial} \phi = -\frac{1}{2} \Omega^{-1} \partial_{\underline{m}} \Omega \Gamma^m - \frac{1}{2} \Omega^{-1/2} \partial_{\underline{y}} \Omega \Gamma^y, \\ \not{G}^{(2)} = \pm 2m \Gamma^{0y}. \end{array} \right. \quad (18)$$

Substituting into the dilatino supersymmetry rule we find

$$-\frac{1}{2}\Omega^{-1}\partial_{\underline{m}}\Omega\Gamma^m\left[1\mp\Gamma^{0y}\Gamma_{11}\right]\epsilon-\frac{1}{2}\Omega^{-1/2}\partial_{\underline{y}}\Omega\Gamma^y\epsilon+\frac{i}{4}m\Omega^{-1/2}\left[5\mp3\Gamma^{0y}\Gamma_{11}\right]\epsilon=0. \quad (19)$$

The first term cancels if we impose

$$\frac{1}{2}\left[1\mp\Gamma^{0y}\Gamma_{11}\right]\epsilon=0, \quad (20)$$

which is the condition satisfied by the Killing spinor of the fundamental string. This operator is a projector and therefore has eigenvalues 1 or 0. The trace is 16, one half of the trace of the identity and therefore this condition breaks a half of the supersymmetries. Using this condition also in the third term we get

$$-\partial_{\underline{y}}\Omega\Gamma^y\epsilon+im\epsilon=0, \quad (21)$$

which is solved by $\alpha = \mp 1$ and

$$m\frac{1}{2}\left[1\mp i\Gamma^y\right]\epsilon=0, \quad (22)$$

which is the condition satisfied by the Killing spinor of a D8-brane. For analogous reasons, this second condition breaks a half of the supersymmetries for $m \neq 0$. These two projectors commute and therefore both conditions can be fulfilled simultaneously. Since the trace of the product of both projectors is 8, 1/4 of the supersymmetries are preserved.

The gravitino equation also vanishes if the Killing spinor is

$$\epsilon=\Omega^{1/4}\epsilon_0, \quad (23)$$

where ϵ_0 is a constant spinor satisfying the above constraints.

Now, observe that if the Killing spinor is an eigenspinor of the fundamental string and D8-brane projectors, then it obeys automatically

$$\frac{1}{2}\left[1\mp i\Gamma^0\Gamma_{11}\right]\epsilon=0, \quad (24)$$

which is the condition of the D0-brane Killing spinor. This may seem a bit surprising since $C^{(1)}$ is trivial (unless y is a compact coordinate). However, its field strength $G^{(2)}$, which is the meaningful quantity is not trivial.

For all these reasons one can identify this solution with the intersection of fundamental string and a D8-brane over a D0-brane⁷.

⁷A string solution to a class of massive supergravity theories was recently given in [20]. However, the mass parameter in that model is of NSNS type and there is no mass term for B . Thus it cannot describe the fundamental string of the massive type IIA theory.

3 Massive D6-Brane

This solution is given by

$$\left\{ \begin{array}{l} ds^2 = \Omega^{-1/2} (dt^2 - d\vec{y}_6^2) - \Omega^{1/2} d\vec{x}_3^2, \\ B_{\underline{mn}} = \mp \frac{m}{3} \epsilon_{mnp} x^p, \\ B^{(6)}_{\underline{ty}^2 \dots \underline{y}^6} = \pm m y^1, \\ C^{(7)}_{\underline{ty}^1 \dots \underline{y}^6} = \pm (\Omega^{-1} - 1), \\ e^{-2\phi} = \Omega^{3/2}, \end{array} \right. \quad (25)$$

where

$$\left\{ \begin{array}{l} \vec{y}_6 = (y^1, \dots, y^6) = (y^i), \\ \vec{x}_3 = (x^1, x^2, x^3) = (x^m), \\ \partial_{\underline{m}} \partial_{\underline{m}} \Omega = -m^2, \\ \partial_{y^1} \Omega = \alpha m. \end{array} \right. \quad (26)$$

Some remarks are necessary:

1. As $C^{(1)}$ in the massive string case, $B^{(6)}$ is pure gauge but we have introduced it only for the sake of consistency.
2. B is *not pure* gauge. A non-trivial $C^{(7)}$ (necessary for a D6-brane) implies a non-trivial $H^{(7)}$ and, by Hodge duality, a nontrivial H and a non-trivial B . This (plus the constraints of unbroken supersymmetry) will give support to the interpretation that there is a solitonic 5-brane in the intersection.
3. The coordinate y^1 has been chosen for simplicity but any other direction in the D6-brane worldvolume (coordinates (t, y^i)) would do as direction orthogonal to the solitonic 5-brane and D8-brane.
4. Again, the function Ω consists of three pieces: a piece linear in y^1 (the coordinate orthogonal to the solitonic 5-brane and the D8-brane), a piece quadratic in \vec{x}_3 (which are interpreted as worldvolume coordinates of the D8-brane, orthogonal to both the solitonic 5-brane and the D8-brane) and a harmonic function of \vec{x}_3 :

$$\Omega = \alpha m y^1 - \sum_p M_p x^p x^p + H(\vec{x}_3), \quad \sum_p M_p = \frac{1}{2} m^2, \quad \partial_{\underline{m}} \partial_{\underline{m}} H = 0. \quad (27)$$

Thus, it can describe, in principle, several objects in equilibrium.

5. In the massless limit $\Omega(y^1, \vec{x}_3) = H(\vec{x}_3)$ and the right choice of H it is just the D6-brane solution.
6. We have a solution for any value of the constant α . However, only for $\alpha = \mp 1$ the solution is supersymmetric. Actually, one finds that (for those values of α) the Killing spinor is

$$\epsilon = \Omega^{-1/8} \epsilon_0, \quad (28)$$

where ϵ_0 is a constant spinor which satisfies

$$\begin{cases} \frac{1}{2} (1 \mp i\Gamma^{01\dots 6}) \epsilon_0 = 0, \\ m_{\frac{1}{2}} [1 \mp i\Gamma^y] \epsilon_0 = 0. \end{cases} \quad (29)$$

If both equations are satisfied, then the following equation is satisfied

$$\frac{1}{2} [1 \pm \Gamma^{02\dots 6}] \epsilon = 0, \quad (30)$$

which is the condition satisfied by the solitonic 5-brane Killing spinor.

It is reasonable to identify these solution with the intersection of a D6- and D8-brane over a solitonic 5-brane.

4 Conclusion

We have presented two new supersymmetric solutions of massive type IIA SUEGRA which have interpreted as the intersection of a string and a D6-brane with a D8-brane over a D0-brane and a solitonic 5-brane respectively. They have a number of remarkable features some of them quite unusual: the massless limit exists (this is the limit in which the D8-brane is removed) but the other limits do not exist. The fact that they consist of 3-objects but 1/4 of the supersymmetries is preserved has also been observed in other contexts. Furthermore, the string solutions is completely localized and depends on all coordinates except on time.

It would be interesting to find other solutions of massive type IIA SUEGRA and to study their possible 11-dimensional origin. Work on this direction is in progress.

Acknowledgments

The authors wish to thank E. Álvarez, H.J. Boonstra, R. Emparán for fruitful discussions. The work of P.M. has been partially supported by the European Union under contract number ERBFMBI-CT96-0616. The work of B.J. and T.O. has been supported by the

European Union TMR program FMRX-CT96-0012 *Integrability, Non-perturbative Effects, and Symmetry in Quantum Field Theory*. The work of T.O. has also been supported by the Spanish grant AEN96-1655.

References

- [1] J. Polchinski, *Phys. Rev. Lett.* **75** (1995) 4724-4727.
- [2] P. Howe and P.C. West, *Nucl. Phys.* **B238** (1984) 181-219.
- [3] M. Huq and M.A. Namazie, *Class. Quantum Grav.* **2** (1985) 293-308.
- [4] F. Giani and M. Pernici, *Phys. Rev.* **D30** (1984) 325.
- [5] J. Polchinski and E. Witten, *Nucl. Phys.* **B460** (1996) 525.
- [6] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Nucl. Phys.* **B470** (1996) 113-135.
- [7] L.J. Romans, *Phys. Lett.* **169B** (1986) 374.
- [8] E. Bergshoeff, Y. Lozano and T. Ortín, *Nucl. Phys.* **B518** (1998) 363.
- [9] E. Bergshoeff and J.P. van der Schaar, Report UG-10/98 and [hep-th/9806069](#).
- [10] P. Meessen and T. Ortín, *An $Sl(2, Z)$ Multiplet of Nine-Dimensional Type II Supergravity Theories*, Report IFT-UAM/CSIC-98-3 and [hep-th/9806120](#). (To be published in *Nuclear Physics B*.)
- [11] C.M. Hull, *Nucl. Phys.* **B509** (1998) 216-251.
- [12] C.M. Hull, *Massive String Theories From M-Theory and F-Theory*, Report QMW-PH-98-36, LPTENS 98/32 and [hep-th/9811021](#).
- [13] G. Papadopoulos and P.K. Townsend, *Phys. Lett.* **B380** (1996) 273-279.
- [14] K. Behrndt, E. Bergshoeff and B. Janssen, *Phys. Rev.* **D55** (1997) 3785-3792.
- [15] J.P. Gauntlett, D.A. Kastor and J. Traschen, *Nucl. Phys.* **B478** (1996) 544-560.
- [16] A. Dabholkar, G.W. Gibbons, J. Harvey and F. Ruiz-Ruiz, *Nucl. Phys.* **B340** (1990) 33.
- [17] T. Sato, *Phys. Lett.* **B441** (1998) 105-115.
- [18] M.R. Douglas, *Branes within Branes*, Report RU-95-92 and [hep-th/9512077](#).
- [19] M.B. Green, C.M. Hull and P.K. Townsend, *Phys. Lett.* **B382** (1996) 65.
- [20] H. Singh, *Phys. Lett.* **B419**(1998), 195.